Section 1.3 Continuity (Minimum Homework: 1, 3, 5, 7, 9, 11, 13, 17, 19, 25, 27)

Continuity is a simple concept. A continuous function is simply a function is a function you can draw without taking your pencil off the paper. If a function has no breaks or gaps, we say the graph of the function is continuous everywhere.

It is easier to describe where a function is discontinuous (not continuous) as opposed to where a function is continuous.

In this section we will find the values of $x = a$ where a function discontinuous.

We will assume it is continuous at every value of $x = a$ other than the values where it is discontinuous (not continuous.)

A function **IS CONTINOUS** at a value of $x = a$ if all three rules are satisifed on the graph of the function for that value of $x = a$.

1. The function is defined at $x = a$;
2. The limit of the function as *x* approaches *a* exists;
3. The value of the limit equals the function value at $x = a$.

The problems in this section will ask us to identify the values of $x = a$ where a function is discontinuous.

A function is discontinuous at $x = a$ if 1 or more of the rules are violated.

To determine where a graph is discontinuous we will:

- Identify the values of x where there is a hole or asymptote (break in the graph).
- Then we will proceed to see which if any of the rules for continuity are violated at the value of x.
- We will stop checking once a rule is violated and write that the function is discontinuous at the value of $x = a$

How to violate a rule:

Rule 1: The function is defined at $x = a$;

This rule is violated if the function is not defined at $x = a$ which happens when:
i) There is a vertical asymptote at $x = a$
ii) The only points marked above $x = a$ are marked with an open circle.

If rule 1 is satisfied proceed to rule 2.

Rule 2: The limit of the function as *x* approaches *a* exists

This rule is violated when the limit does not exist at $x = a$.
For our checking this rule will be violated when graph goes in one point and out another.
(asymptotes at $x = a$,are also a violation of this rule, we won't get to this step if there is an asymptote)

If both rule 1 and rule 2 are satisfied proceed to rule 3

Rule 3: The value of the limit equals the function value at $x = a$.

We won't get to this rule if the limit doesn't exist. The limit has to exist for us to get to step 3.

This rule will be violated if the value of the function does not equal the value of the limit. Specifically we will say this rule has been violated provided the limit exists at a point marked with an open circle and there is a point marked with a closed circle with the same value of x.

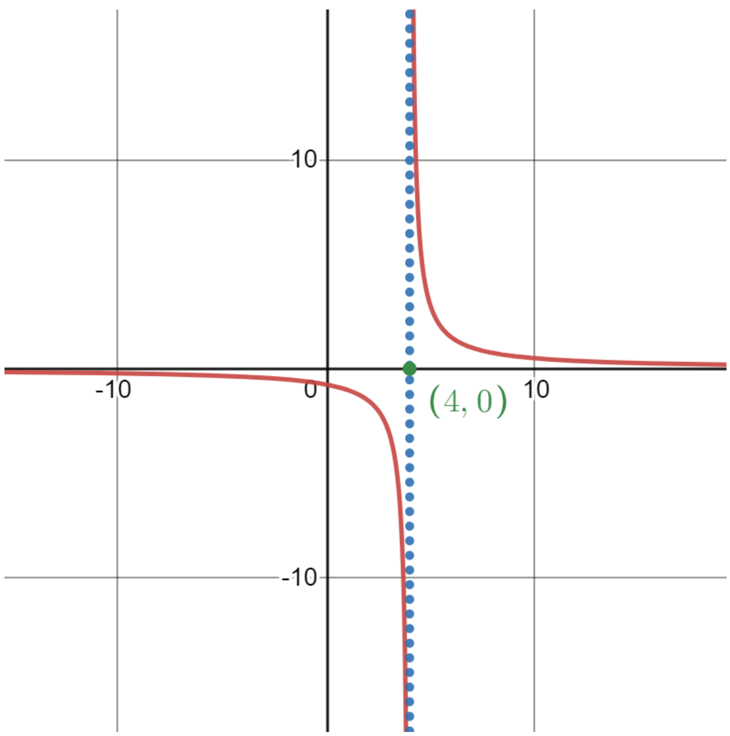
Example: Find all values of x = a where the function is discontinuous. State the first reason that makes the function discontinuous for the value of $x = a$.

The only break in the graph is at $x = 4$ and it is the only value of x where the graph is discontinuous.

Reason 1: The function is NOT defined at $x = 4$ as there is a vertical asymptote at $x = 4$ (reason 1 is violated)

I will stop checking as I have found a rule violation

- **Answer: Is discontinuous at $x = 4$ for reason 1**



Example: Find all values of x = a where the function is discontinuous. State the first reason that makes the function discontinuous for the value of $x = a$.

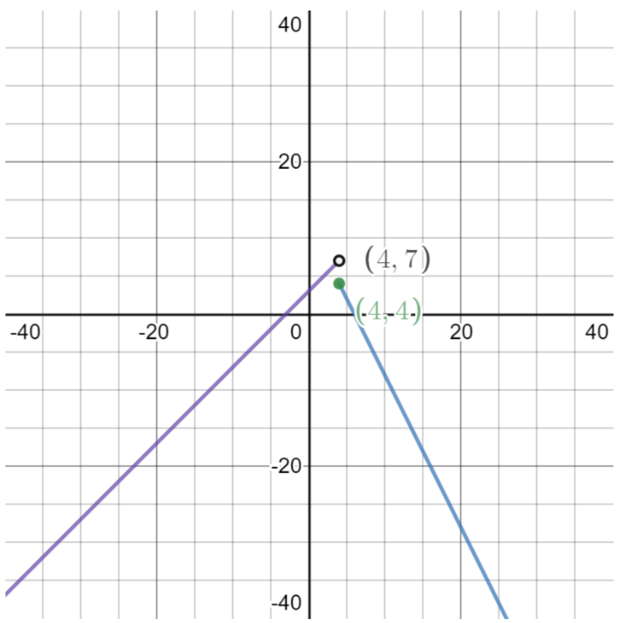
The only break in the graph is at $x = 4$ and it is the only value of x where the graph is discontinuous.

Reason 1: The function is defined at $x = 4$ as there is a solid point marked with an x-coordinate of 4 , so reason 1 is not violated

Reason 2: The limit does NOT exist at $x = 4$ as the graph does not go in and out of the same point at $x = 4$. Reason 2 is violated.

I will stop checking since I have found a rule violation.

- **Answer: discontinuous at $x = 4$ for reason 2**



Example: Find all values of x = a where the function is discontinuous. State the first reason that makes the function discontinuous for the value of $x = a$.

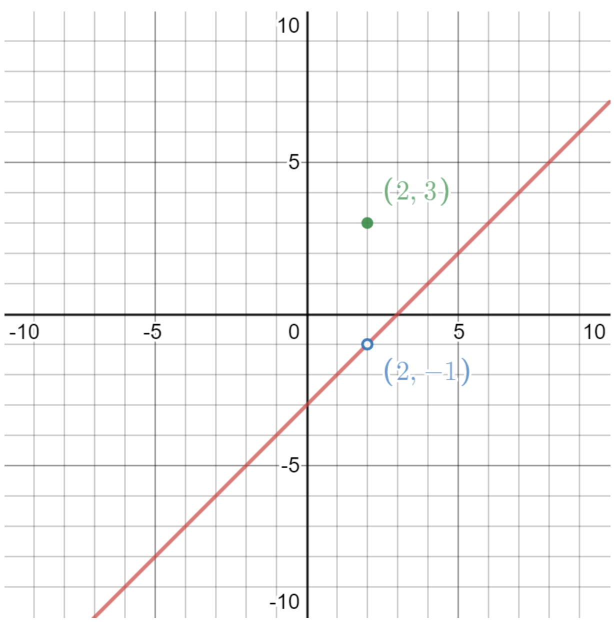
The only break in the graph is at $x = 2$ and it is the only value of x where the graph is discontinuous.

Reason 1: The function is defined at $x = 2$ as there is a solid point marked with an x-coordinate of 4 , so reason 1 is not violated

Reason 2: The limit exists at $x = 2$ as the graph goes in and out of the same point at $x = 2$. Reason 2 is not violated.

Reason 3: The limit exists, but the value of the limit is the y-coordinate of the open hole, and the value of the function is the y-coordinate of the solid hole. Reason 3 is violated.

- **Answer: Is discontinuous at $x = 2$ for reason 3**



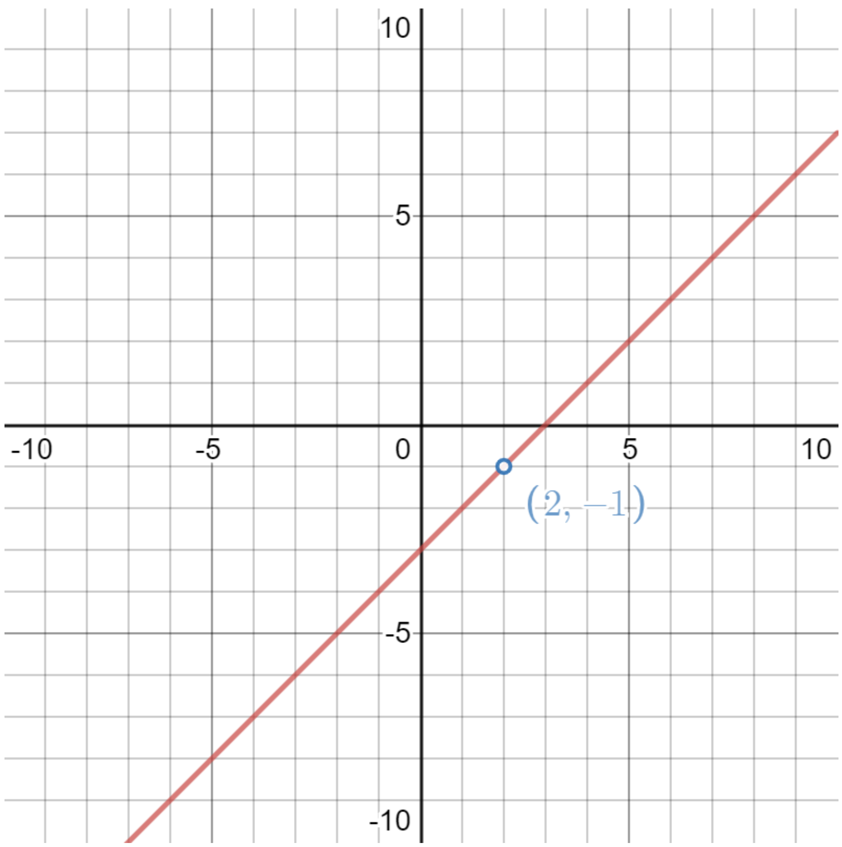
Example: Find all values of x = a where the function is discontinuous. State the first reason that makes the function discontinuous for the value of $x = a$.

The only break in the graph is at $x = 2$ and it is the only value of x where the graph is discontinuous.

Reason 1: The function is NOT defined at $x = 2$ as there is NO solid point marked with an x-coordinate of 2 , so reason 1 is violated

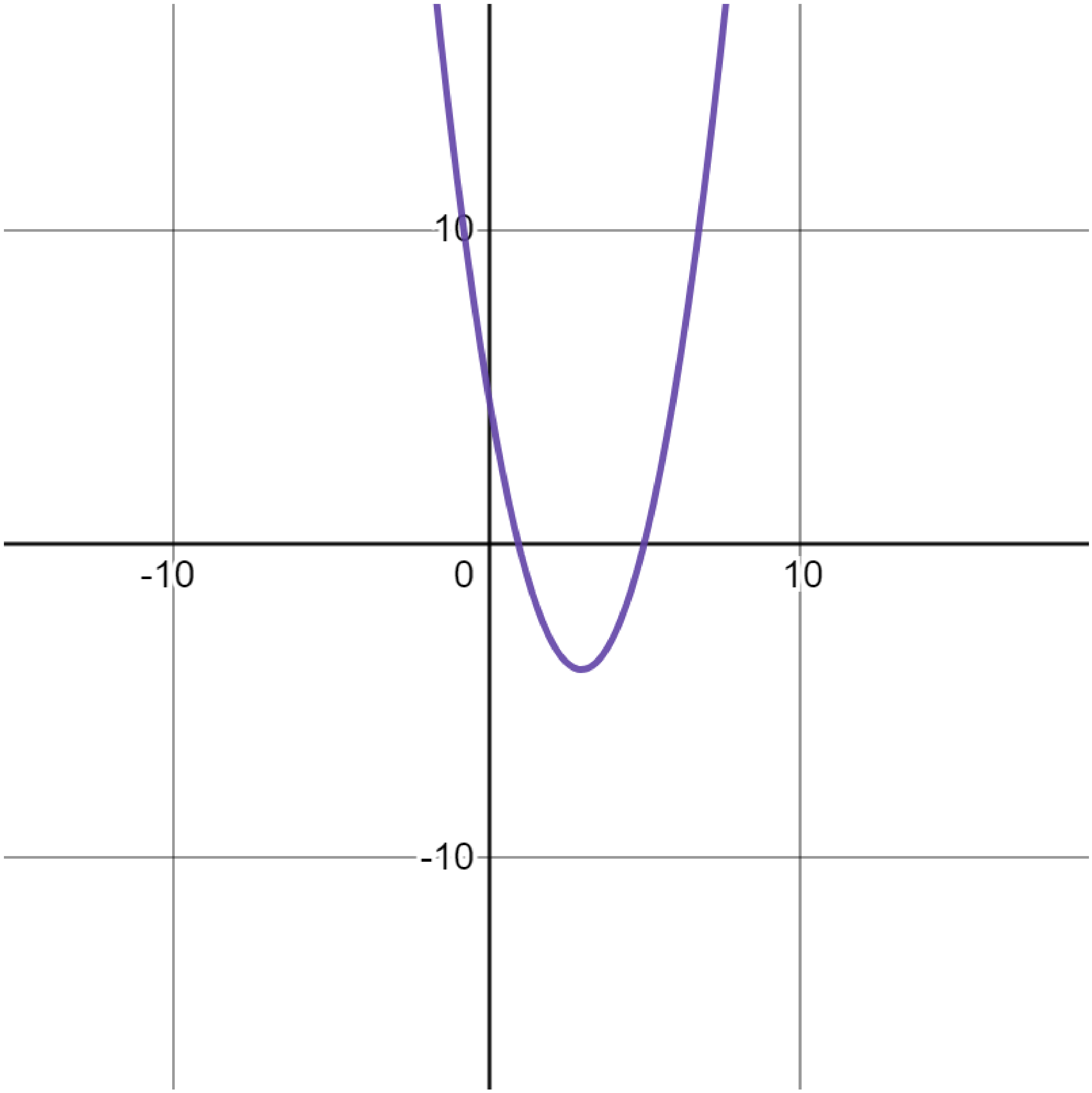
I can stop checking as I have found a violation.

- **Answer discontinuous at $x = 2$ for reason 1**



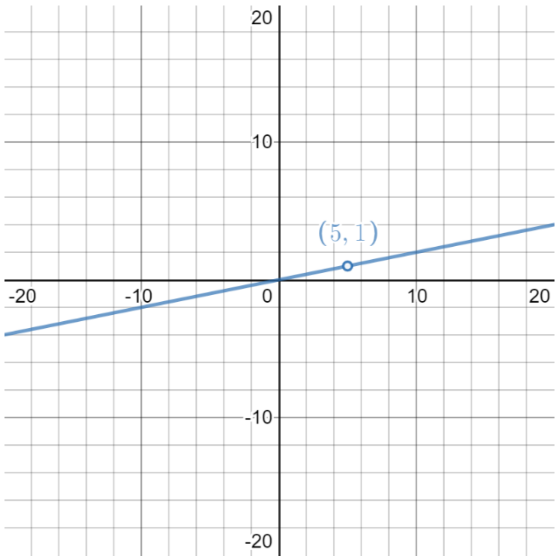
If all there are no values of $x = a$ where a graph of a function is not continuous, we say the function is continuous everywhere.

Here is an example of function that is continuous everywhere.

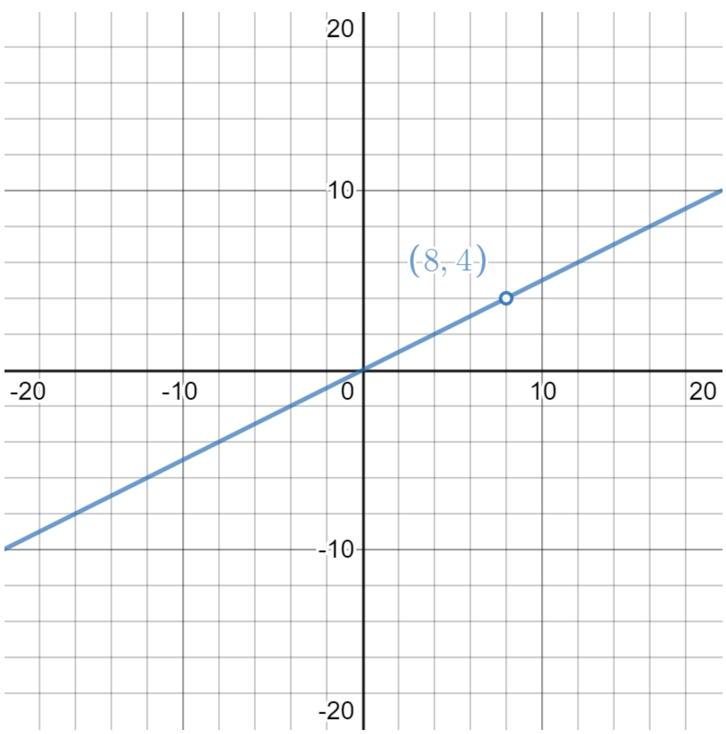


#1-10: Find all values of x = a where the function is discontinuous. State the first reason that makes the function discontinuous for the value of $x = a$.

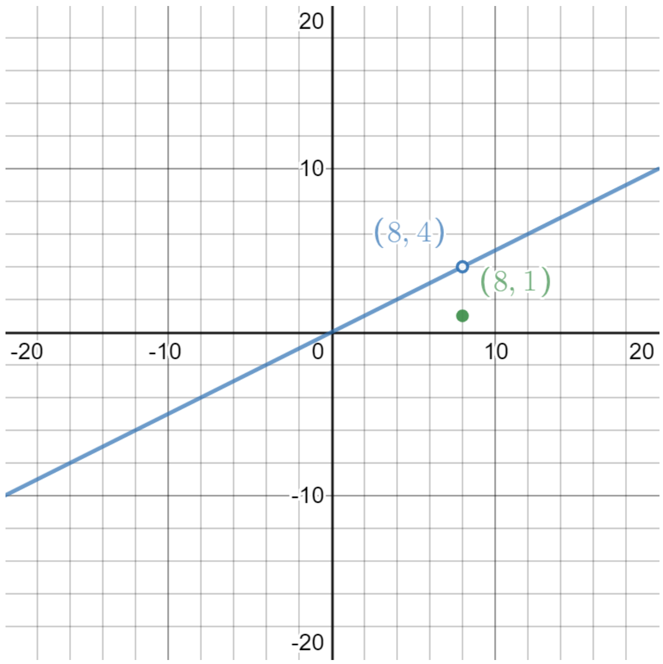
1)



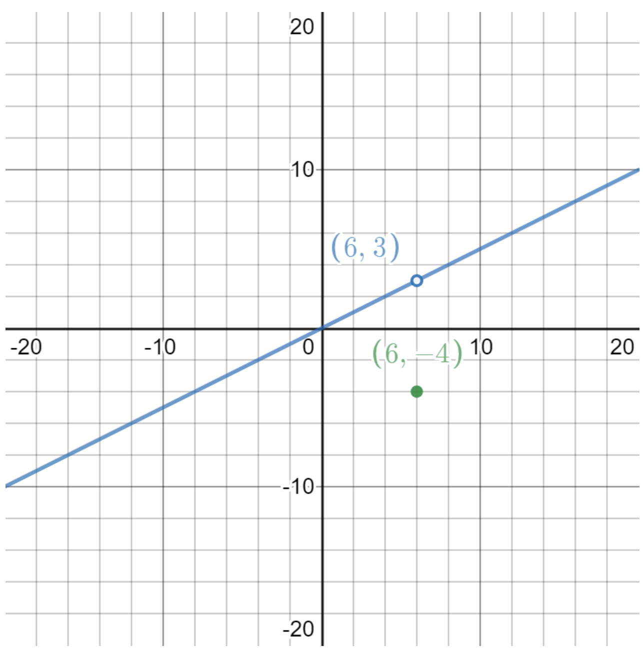
2)



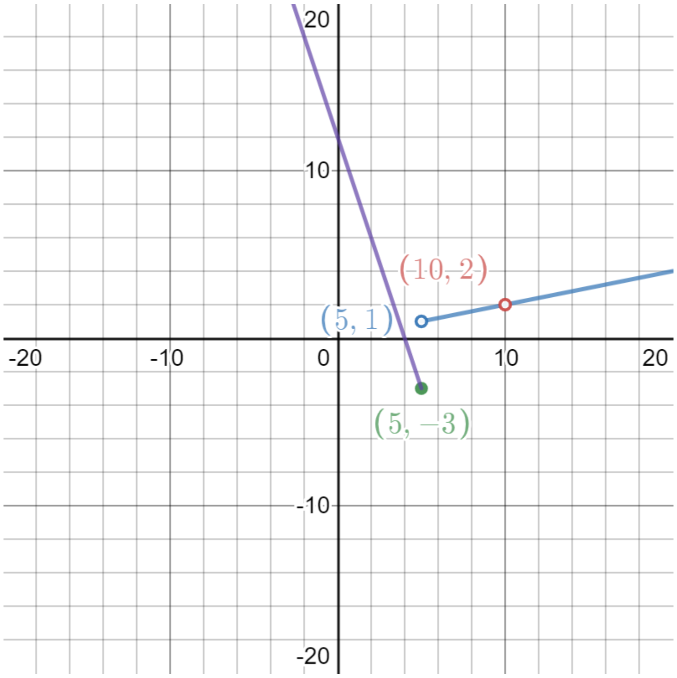
3)



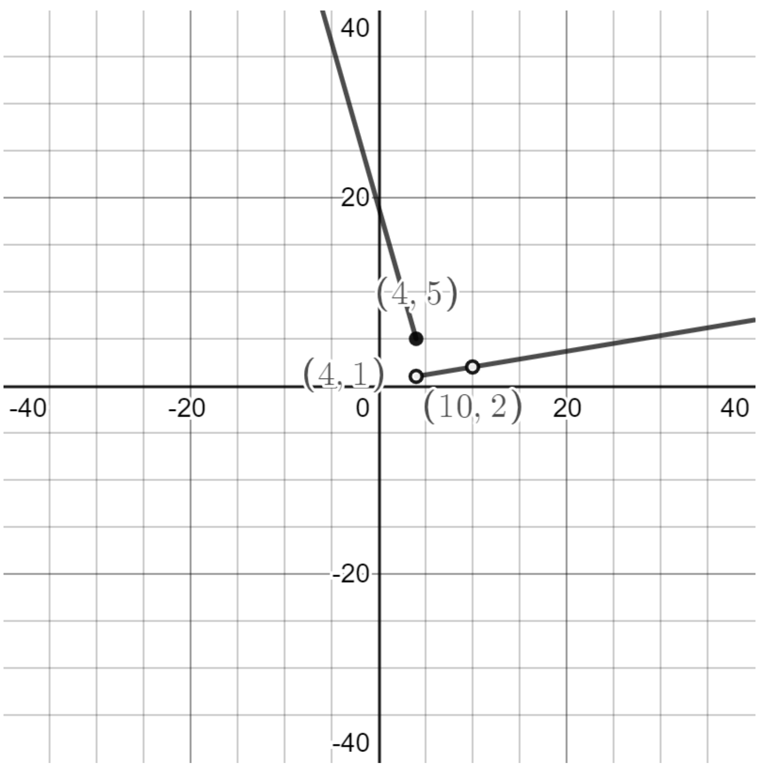
4)



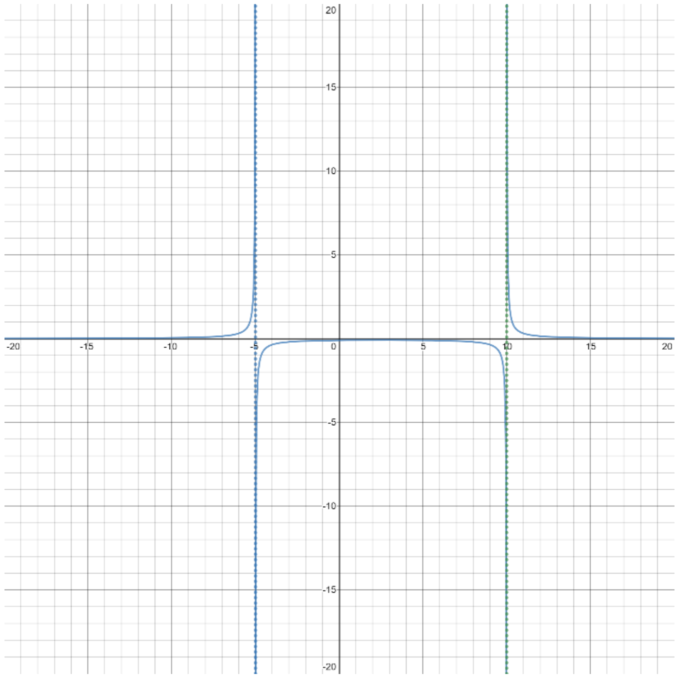
5)



6)

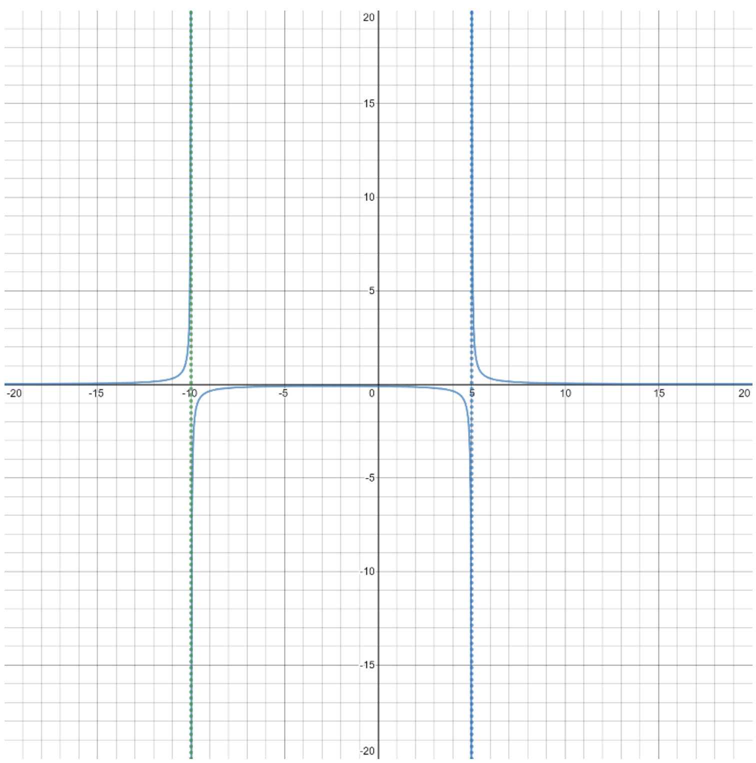


7)



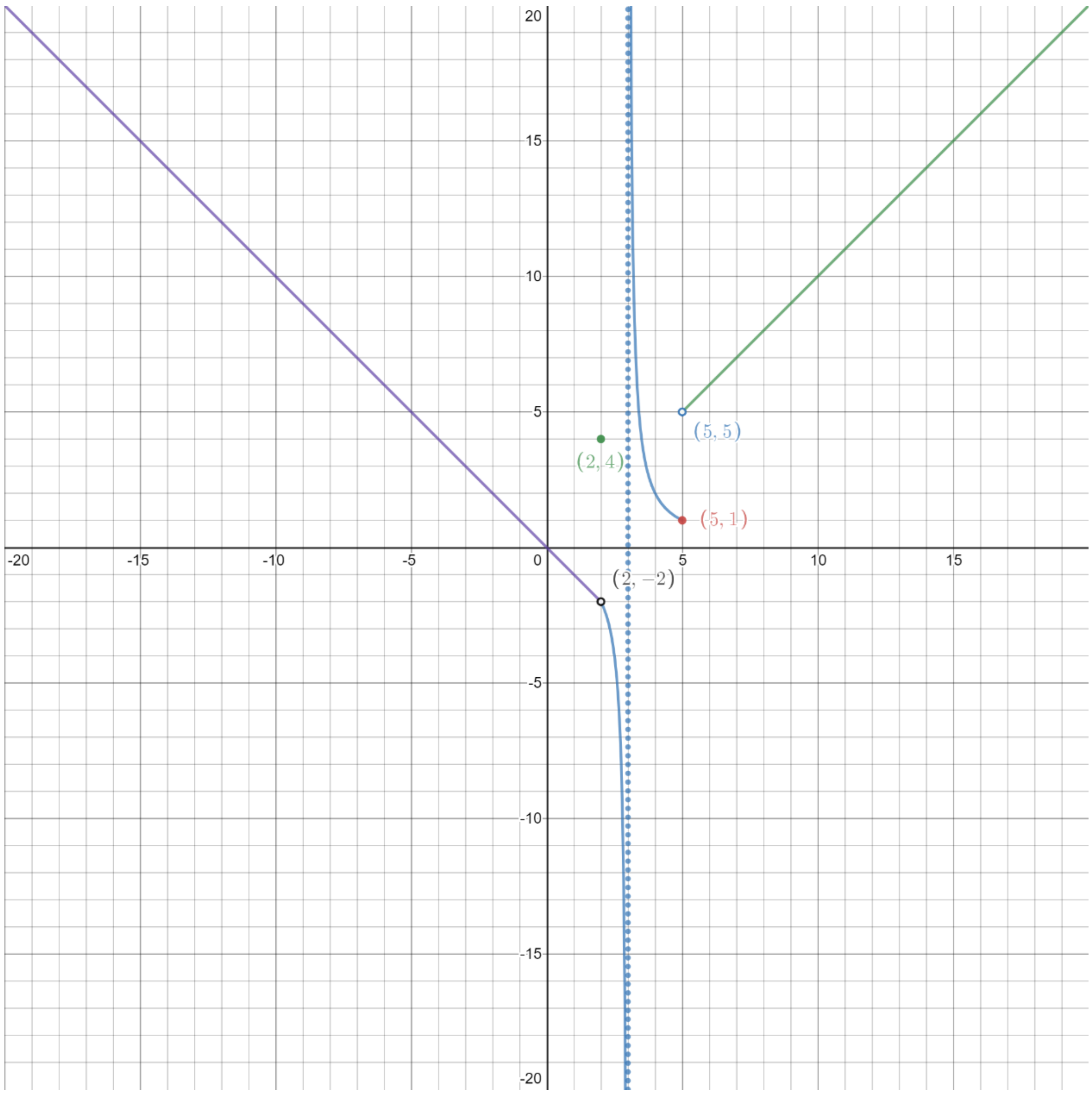
20
15
10
5
-5
-10
-15
-20
-20
-15
-10
-5
0
5
10
15
20

8)

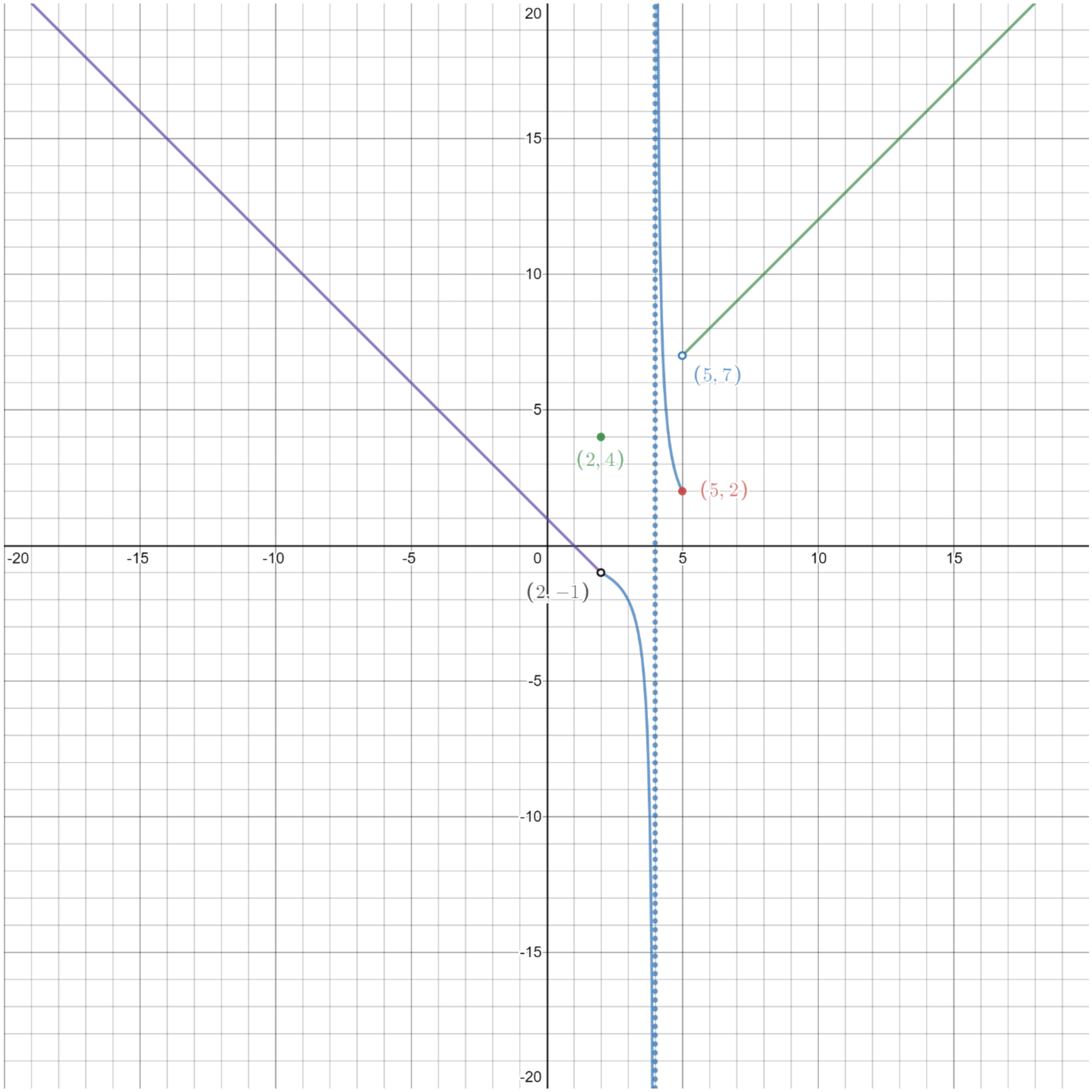


20
15
10
5
-5
-10
-15
-20
-20
-15
-10
-5
0
5
10
15
20

9)



10)



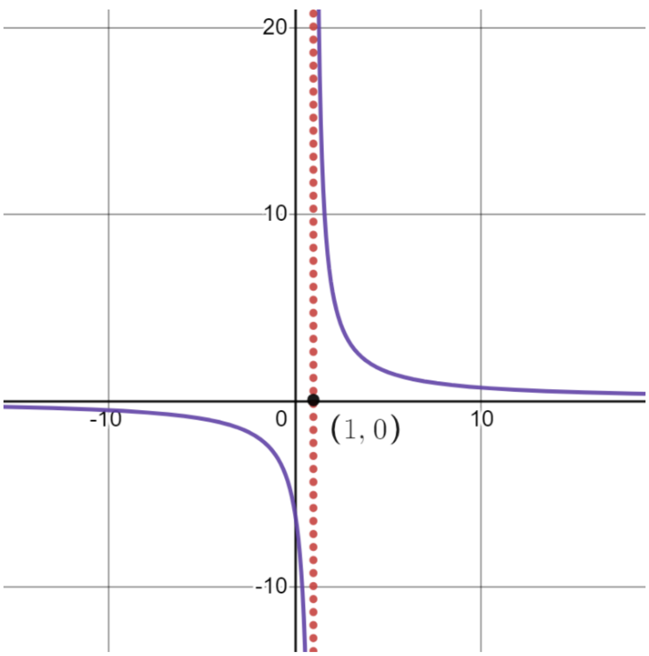
Example: Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.

$f(x) = \frac{6}{x-1}$

The graph of a function will not be continuous at any values of $x = a$ that make the denominator zero since the function is not defined when the denominator equals 0 as the function is not defined for these values of x = a.

$Solve\ x - 1 = 0$
$x - 1 = 0$
$x = 1$

Answer: The function is **NOT CONTINOUS** at $x = 1$



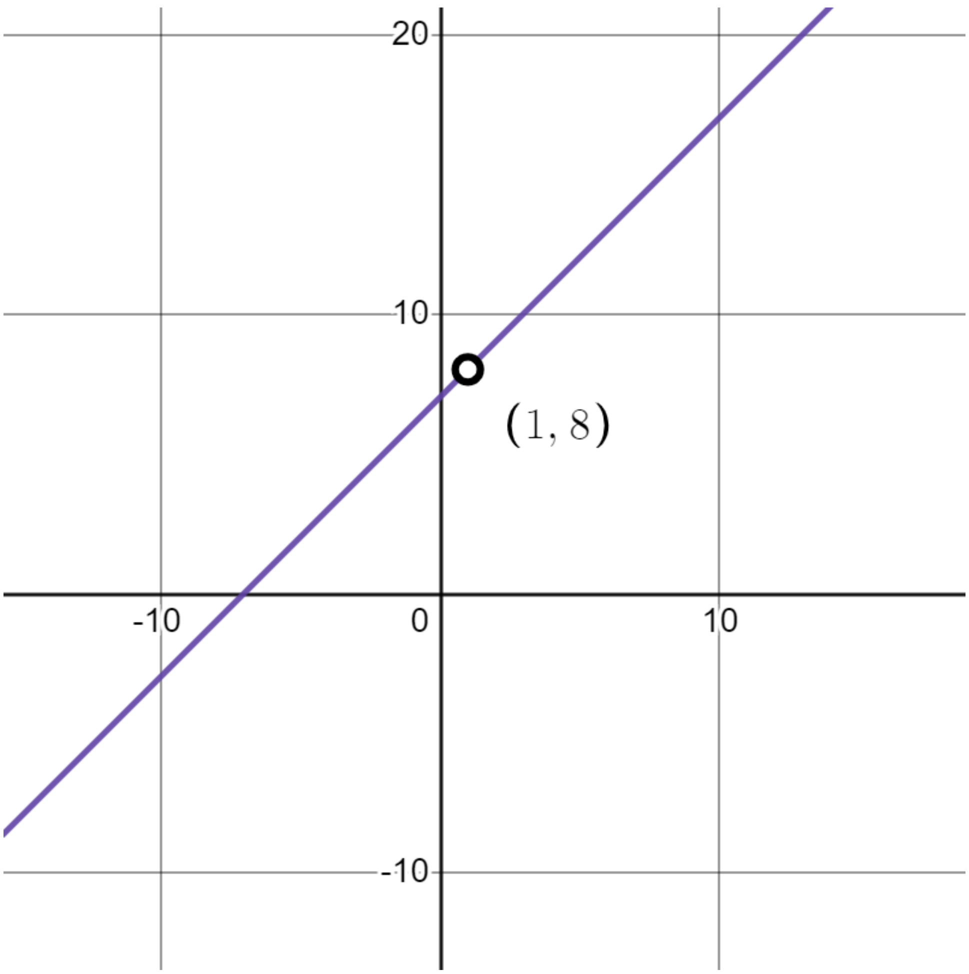
Example: Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.

$f(x) = \frac{x^2+6x-7}{x-1}$

The function is not defined when the denominator equals 0.

$Solve\ x - 1 = 0$

Answer: The function is **NOT CONTINOUS** at $x = 1$



Example: Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.

$$f(x) = \begin{cases} x - 5, & if\ x \le 6 \\ 2x - 9, & if\ x > 6 \end{cases}$$

The only value of $x = a$ where the function may not be continuous is at $x = 6$ as the two pieces that make up the function are polynomials. (Graphs of polynomials are continuous everywhere)

Plug $x = 6$ into both functions.

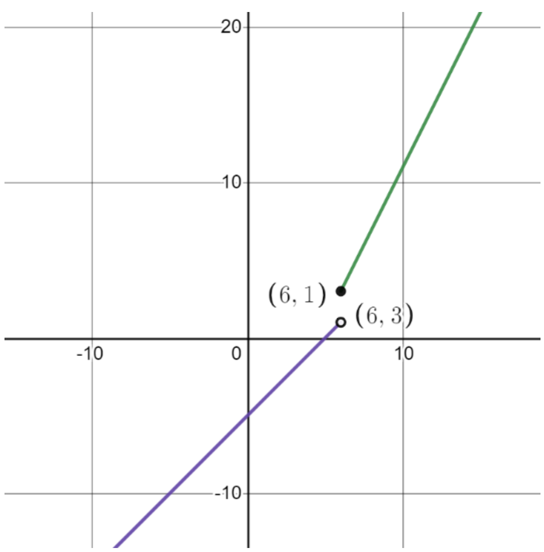
If the values do not equal, limit does not exist at $x = 6$ and the function will not be continuous at $x = 6$ for reason 1.

If the values are equal the limit will exist at $x = 6$ and the function will be continuous everywhere

Top function: $f(6) = 6 - 5 = 1$

Bottom function: $f(6) = 2(6) - 9 = 3$

The function is **not** continuous at $x = 6$ (this would be reason 2)



Example: Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.

$$f(x) = \begin{cases} x - 3, & if\ x \le 6 \\ 2x - 9, & if\ x > 6 \end{cases}$$

The only value of $x = a$ where the function may not be continuous is at $x = 6$ as the two pieces that make up the function are polynomials. (Graphs of polynomials are continuous everywhere)

Plug $x = 6$ into both functions.

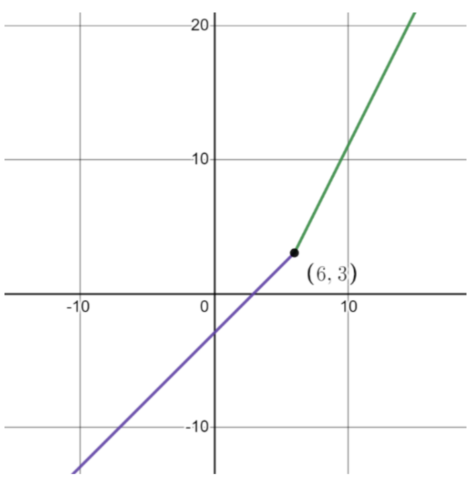
If the values do not equal, limit does not exist at $x = 6$ and the function will not be continuous at $x = 6$.

If the values are equal the limit will exist at $x = 6$ and the function will be continuous everywhere

Top function: $f(6) = 6 - 3 = 3$

Bottom function: $f(6) = 2(6) - 9 = 3$

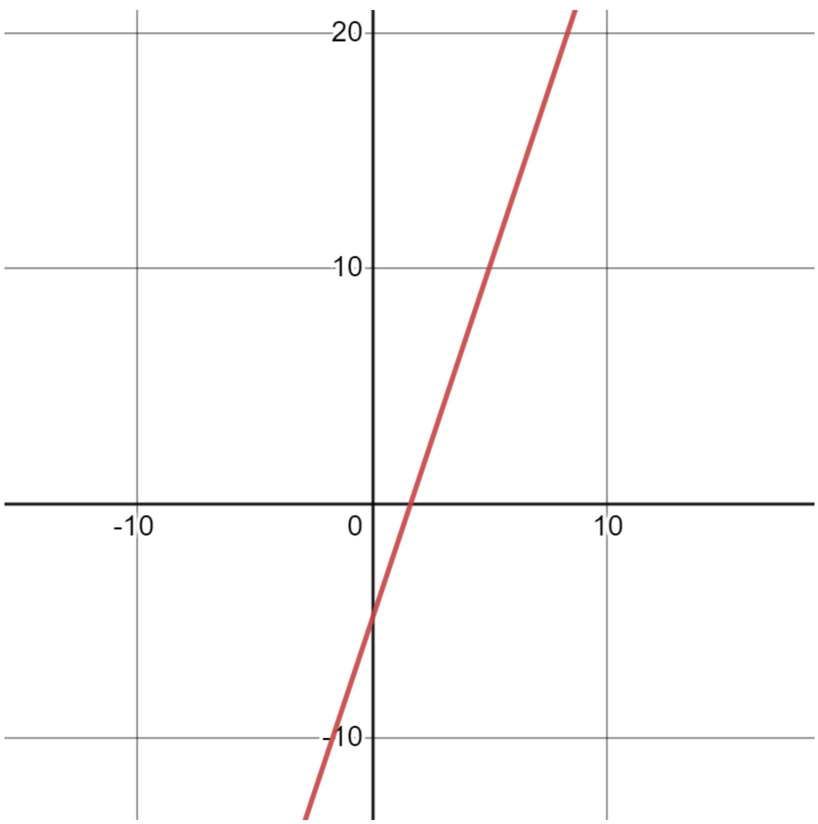
Answer: The function is **CONTINOUS** everywhere.



Example: Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.

$f(x) = 3x - 5$

Answer: The function is **CONTINOUS** everywhere. Unfortunately, there is no Algebra to show this, the graph of a polynomial is continuous everywhere.



#11-30: Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.

11) $f(x) = \frac{x-3}{x+4}$

12) $f(x) = \frac{x+1}{x-5}$

13) $f(x) = \frac{x^2+7x+12}{x+3}$

14) $f(x) = f(x) = \frac{x^2+3x+2}{x+1}$

15) $f(x) = \frac{x^2-4}{x+2}$

16) $f(x) = \frac{x^2-9}{x+3}$

17) $f(x) = \frac{5}{x^2+3x+2}$

18) $f(x) = \frac{7}{x^2-2x-3}$

19) f(x) = 2x – 6

20) f(x) = 3x – 2

21) f(x) = x^2 + 6x – 7

22) f(x) = x^2 – 4x – 5

23) $f(x) = \begin{cases} x+3, & if\ x \le 6 \\ 2x, & if\ x > 6 \end{cases}$

24) $f(x) = \begin{cases} 6x, & if\ x \le 2 \\ 2x+1, & if\ x > 2 \end{cases}$

25) $f(x) = \begin{cases} x-3, & if\ x \le 5 \\ 2x-9, & if\ x > 5 \end{cases}$

26) $f(x) = \begin{cases} x-3, & if\ x \le 4 \\ 2x-9, & if\ x > 4 \end{cases}$

27) $f(x) = \begin{cases} x+6, & if\ x \le 6 \\ 2x, & if\ x > 6 \end{cases}$

28) $f(x) = \begin{cases} 6x, & if\ x \le 2 \\ 2x+8, & if\ x > 2 \end{cases}$

29) $f(x) = \begin{cases} x-3, & if\ x \le 5 \\ 2x-8, & if\ x > 5 \end{cases}$

30) $f(x) = \begin{cases} x-3, & if\ x \le 4 \\ 2x-7, & if\ x > 4 \end{cases}$